

D-Particle Dynamics and The Space-Time Uncertainty Relation

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Abstract

We argue that the space-time uncertainty relation of the form $\Delta X \Delta T \gtrsim \alpha'$ for the observability of the distances with respect to time, ΔT , and space, ΔX , is universally valid in string theory including D-branes. This relation has been previously proposed by one (T.Y.) of the present authors as a simple qualitative representation of the perturbative short distance structure of fundamental string theory. We show that the relation, combined with the usual quantum mechanical uncertainty principle, explains the key qualitative features of D-particle dynamics.

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It is often stated that in the fundamental string theory there exists a minimum length of order of $\sqrt{\alpha'} \equiv \ell_s$ beyond which we cannot probe the structure of space-time and hence the ordinary concept of space-time ceases to be meaningful. This comes about from the properties of string amplitudes in the high-energy limit [1] [2] and also in the high temperature limit [3]. Such a statement is indeed quite natural when we have only the ordinary string states as possible probes for short distances, since string states themselves have an intrinsic extension of the order of length ℓ_s .

Recently, however, we understood that string theory in fact allows a variety of objects of various dimensions as solitonic excitations and that they are bound to play crucial roles in nonperturbative formulations of string theory. In particular, we have even point-like objects called D0-branes [4] or D-particles. Recent studies [5] [6] [7] [8] [9] [10] [11] [12] of D-particle dynamics revealed the possibility of probing the distance scales of 11D Planck scale of the order $g_s^{1/3}\ell_s$, the natural scale of the M-theory [13], which is indeed much shorter than the string scale ℓ_s for weak string coupling ¹. Therefore, the usual folklore statement quoted above must now be reconsidered. If we remember that the string scale represents the unique fundamental constant of Nature in the natural unit $c = \hbar = 1$, its precise significance must certainly be clarified. The purpose of the present note is to remark that a simple space-time uncertainty relation [15] proposed in 1987 by one of the present authors is an appropriate interpretation for the meaning of the string scale ℓ_s , since it is universally valid both for ordinary string scattering and D-particle dynamics.

Let us begin with briefly recalling the arguments of reference [15] which motivates the space-time uncertainty relation. Consider a high energy scattering of arbitrary objects whose interactions are mediated by strings. If the energy scale is of order E , the smallest time scale probed by this scattering event is of order $\Delta T \sim \frac{1}{E}$. Now, what is the typical spatial length scale probed by this scattering event? If both the scattering objects and their interactions

¹ The importance of shorter length scales in string theory has been suggested earlier in [14].

were described by usual local field theories neglecting quantum gravity, we would be allowed to state that it is determined by the typical wave length of the objects, namely, $\frac{1}{E}$ for sufficiently high energies. Hence, in principle, we would have no limitation for probing the short-distance scale, provided we neglect quantum gravity. If on the other hand the interactions are mediated by fundamental strings, high energies do not necessarily imply that the typical spatial scale is given by the wave length of the scattering objects, since higher energies dominantly cause larger fluctuations with respect to string excitations during interactions than with respect to the center of mass motion because of the huge degeneracy of string excitation modes. It is easy to see [15] [16] that the typical (smeared-out) spatial extension ΔX of strings with energy E is of order $\Delta X \sim \ell_s^2 E$. This implies the simple relation for the indeterminacies of the space and time lengths

$$\Delta X \Delta T \gtrsim \ell_s^2, \quad (1)$$

which we call the space-time uncertainty relation. In reference [15], this relation was proposed as a natural space-time representation of the *st*-duality properties of string scattering amplitudes. As discussed later in reference [16], it can also be derived as a direct consequence of the world-sheet conformal invariance.

From the view point of the space-time uncertainty relation, the usual argument [17] for the minimal length essentially amounts to assuming that the observable length is the average of the spatial and time distances; then we would have the lower bound $\frac{\Delta X + \Delta T}{2} \geq \ell_s$. Clearly, however, what is the dominant scale measured by scattering experiments depends on which kinematical regions we are interested in. For example, a high-energy low-momentum transfer (peripheral) scattering experiment can probe small time scale, but the spatial scale is not necessarily small. Thus, it corresponds to $\Delta T \rightarrow 0$ and our relation (1) implies that the spatial length scale grows as $\frac{\ell_s^2}{\Delta T} \sim \ell_s^2 E$ as the laboratory energy E increases. By adapting the string-bit argument due to Susskind [18], this longitudinal length scale of a string, growing linearly with energy, leads to the Regge intercept $\alpha(0) = 2$ of the probability

amplitude $A(E) \sim E^{\alpha(t)-1}$, in conformity with the existence of a graviton ². On the other hand, a high-energy fixed-angle scattering as studied in [2] tries to probe short distances with respect to both space and time. In this case, since the relation (1) shows there is no such degrees of freedom, the amplitude vanishes exponentially in the high-energy limit.

As long as we only use the strings as probes, it seems difficult to imagine a scattering experiment which makes it possible to measure directly the region $\Delta X \rightarrow 0$, because of the intrinsic extension of the string. In references [15] [16], it was suggested to interpret the relation (1) in the limit $\Delta X \rightarrow 0$ as an explanation why it is possible to treat the asymptotic string states, propagating infinitely long time $\Delta T \rightarrow \infty$, as *local* external fields in the sigma-model approach to world-sheet string theory. The asymptotic states correspond to the s -channel poles. Combined with the Regge-pole exchange picture for the limit $\Delta T \sim \frac{1}{\Delta X} \rightarrow 0$, the relation (1) is thus interpreted as a natural space-time interpretation of [Regge-pole] \leftrightarrow [resonance-pole] duality. Now it is clear that the D0-branes are ideal objects for the purpose of directly probing the short spatial length scale and testing the relation (1) beyond such formal arguments. Fortunately, there already appeared several works cited above which studied the dynamics of D0-branes in the low-energy limit. In the following, we show that all the results so far are consistent with the relation (1) and the most crucial feature behind these results can be naturally understood on the basis of (1).

First of all we note that the slow velocity limit studied in these works is just appropriate for probing the small ΔX regions where (1) implies that ΔT grows. Moreover, it seems fairly clear that the space-time uncertainty relation just conforms to the basic principles emphasized in reference [10] that the leading singular behavior in the short spatial distance limit is determined by the IR behavior of brane world-volume quantum theory. In fact this statement is a direct consequence of the duality between s -and- t channels which was

² Here, t is the invariant momentum transfer whose dependence is basically determined by the effective transverse size of the string.

nothing but the original interpretation of the relation (1) as discussed above. In this sense, our discussion will provide a shortcut to understanding some of the basics for previous results on the short distance behavior of D-particle dynamics.

Now let us consider the scattering of two heavy D-particles of mass $m = \frac{1}{g_s \ell_s}$ with slow typical velocity v . If we assume that there is a limitation for the meaningful space-time lengths in the form (1), what is the smallest possible spatial length scale b probed by this scattering? Let

$$b \sim v^\eta \ell_s.$$

The typical time length of the scattering is $t = \frac{b}{v}$. Substituting these relations to (1) with $\Delta X \sim b, \Delta T \sim t$, we must have $\eta = 1/2$,

$$b \sim \sqrt{v} \ell_s. \tag{2}$$

This length scale first appeared in [6], where an annulus amplitude of open strings is computed, and was further analyzed in detail in [10] using the effective world line theories [19] (0 + 1-dimensional super Yang-Mills matrix quantum mechanics) of D-particles. The relation (2) constitutes one of the most crucial relation in all previous discussions of short-distance structure in D-particle dynamics. Following [10], we often call the scale $\sqrt{v} \ell_s$ the stadium size. From the point of view of the effective super Yang-Mills matrix model, the stadium size is the limit for the spatial scale where the Born-Oppenheimer approximation for the coupling between the D-particle coordinates and the short open-string excitation connecting them ceases to be valid. The dynamics of latter corresponding to the off-diagonal part of the adjoint Higgs fields originated from the 10D gauge fields by dimensional reduction is governed by the time scale $\mathcal{O}(\ell_s^2/b)$ while that of the former, the diagonal part of the Higgs, is by $\mathcal{O}(b/v)$. If the Born-Oppenheimer approximation is not valid, we cannot clearly separate the diagonal part as the spatial coordinates of D-particles and hence we must have (2). Our proposal is thus to interpret this result as a consequence of the universal space-time uncertainty relation (1).

Now once the relation (2) is known, the important fact that the characteristic spatial length scale of D-particle dynamics is nothing but the eleven dimensional Planck scale associated with M-theory is understood from the usual uncertainty relation. If the time duration of the scattering is of order $\Delta T \sim \frac{b}{v} \sim v^{-1/2}\ell_s$, usual time-energy uncertainty relation applied for a point-like D-particle implies an uncertainty with respect to the D-particle velocity of order $\Delta v \sim g_s v^{-1/2}$ which leads to the spread of the wave packet of order $g_s v^{-1}\ell_s$ during the time interval $v^{-1/2}\ell_s$. Here we used the fact that the kinetic energy of a D-particle is $\frac{1}{2g_s\ell_s}v^2$ for weak string coupling ³. In order that the minimum length scale $b \sim \sqrt{v}$ be meaningful, b must be larger than this spread. Thus we have the lower bound for the velocity [10]

$$v \gtrsim g_s^{2/3}$$

which leads to the 11D Planck scale $b \sim g_s^{1/3}\ell_s$ of the M theory as a meaningful smallest distance probed by D-particle scattering at low velocities.

From the view point of the effective super Yang-Mills matrix model, the 11D Planck scale is easily understood from a scaling argument [8] [9] which says that coupling constant g_s can be eliminated from the dynamics by making a rescaling, $X_i \rightarrow g_s^{1/3}X_i$ and $t \rightarrow g_s^{-1/3}t$, for the D-particle coordinates X_i and the time t , respectively. We here emphasize a trivial but crucial fact that the *opposite* scalings for the space and time coordinates just conform to a necessary requirement for the validity of the relation (1).

Actually, as pointed out in [10], when we consider a D-particle in the presence of a large number of D4-branes, it becomes possible to probe arbitrary short spatial distance scale. This is basically due to the fact that the D4-branes produce an effective metric for the moduli space of a D0-particle which makes the effective mass of D0-particle much heavier than that in the flat space at short distances: The effective action in the presence of N

³ For the present order estimate, we can neglect the potential energy of order $\mathcal{O}(\frac{v^4}{(\Delta X)^7})\ell_s^6$ which is smaller than the kinetic energy when $\Delta X \gtrsim b$.

coincident parallel D4-branes is given by

$$S_{\text{eff}} = \int dt \left[\frac{1}{2g_s \ell_s} \left(1 + \frac{Ng_s \ell_s^3}{r^3} \right) v^2 + \mathcal{O}(Nv^4 \ell_s^6 / r^7) \right]$$

where r is the distance between the D-particle and D4-branes. According to [10], this metric is likely to be exact without α' and instanton corrections. When the distance r is much shorter than $(Ng_s)^{1/3} \ell_s$, the mass of the D-particle is effectively given by $m \sim N \ell_s^2 / r^3 \gg \frac{1}{g_s \ell_s}$. We can then easily check that the spread of the D-particle wave packet can be neglected during the time $t \sim v^{-1/2} \ell_s$ compared with the stadium size $\sqrt{v} \ell_s$ for large N (if r itself is the stadium size $\sqrt{v} \ell_s$). This allows us to probe arbitrary short lengths with respect to the distance between the D-particle and D4-branes and hence indicates that the singular spatial metric in the D0-particle moduli space is meaningful even in the limit $r \rightarrow 0$. On the other hand, since the time scale grows indefinitely, we cannot talk about the interaction time in any meaningful way in the limit of short spatial distance.

The space-time uncertainty relation (1) in general says that to probe the short spatial distances, inversely large time interval is necessarily required and vice versa. Thus if it is universally valid, we would not be able to introduce the concept of space-time event which is local with respect to both space and time.

Although we expect that the space-time locality is lost eventually in any theory including quantum gravity, the relation (1) suggests a specific manner on how this happens in fundamental string theories. It is very important to see whether this way of expressing the significance of the string constant ℓ_s is useful in the dynamics of more general branes and strings, including D-instantons. Since the interaction between general D-branes is governed by the fundamental strings and the relation (1) originates from the conformal invariance of the fundamental string dynamics, it is reasonable to expect its general validity, if we interpret the relation appropriately. Here, the case of D-instantons (D-1-branes) is very special since we cannot talk about time evolution in their dynamics. It is known [21] that the invariant scattering amplitude of massless string states off a fixed D-instanton for arbitrary energy is simply given by $\frac{1}{t}$ apart from the kinematical factor where t is the invariant t -channel energy,

provided we supply appropriate fermion contribution to cancel possible fermion zero modes. The pole $t = 0$ represents the massless dilaton exchange contribution. Namely, the amplitude for weak string coupling is reproduced by a local field theory without any α' correction for any energy. This is *not* a contradiction to our space-time uncertainty relation, since the above behavior can be interpreted to correspond to the special case where $\Delta T \sim \frac{1}{\Delta X} \sim 0$: $\Delta T \sim 0$ reflects the point-like nature of the D-instanton in space-time, while $\Delta X \rightarrow \infty$ is associated with the long range propagation of a virtual dilaton exchange. Indeed, after the integration over the position of a single D-instanton, we would have $t = 0$. From the view point of (1), the appearance of the long range exchange of massless dilaton is a necessary condition for the existence of the point-like instanton ($\Delta T \sim 0$) contribution. Therefore, the apparent loss of stringy property of the D-instanton-string interaction may be regarded as yet another piece of evidence for the universal nature of the space-time uncertainty relation. Of course, this argument is restricted on one instanton case. The multi-instanton dynamics is much more complicated due to stringy interactions and the integration over the collective coordinates. Further investigations on general D-brane dynamics including instantons from our view point will be useful for clarification towards more precise and general interpretation of the space-time uncertainty relation.

Our discussions so far assumed weak string coupling. However, given that, except for type IIA theory and heterotic $E_8 \times E_8$ theory, the S-duals of string theories are again string theories, it is natural to expect that the relation is valid even at strong string coupling. Note that the string constant ℓ_s can be regarded to be invariant under the S-dual transformations, and therefore the string tension with respect to the correctly rescaled space-time coordinates remain the same as well. In the case of type IIA (which we are mainly concerned with in the present note) and heterotic $E_8 \times E_8$, the dual is the M-theory, whose strong (string) coupling limit is believed to be described by 11D supergravity in the long-distance limit. If the recent interesting conjecture [20] that the microscopic M-theory is described *exactly* by the 0 + 1D Yang-Mills matrix model in the infinite-momentum frame is correct, it is plausible that our uncertainty relation continues to be valid even for strong string coupling,

since the time scale in the Galilean dynamics of the off-diagonal elements is then always given by the difference of the diagonal elements $\frac{\ell_s^2}{|x_i - x_j|}$ for the D-particle coordinates x_i, x_j , which is the basis for the relation (1) from the view point of the super Yang-Mills matrix model as a world line theory of D-particles.

Another relevant question towards a possible generalization of the space-time uncertainty relation would be whether we can have similar relation for spatial domains without including the time length. The appearance of the lower bound $\Delta X \sim b \gtrsim g_s^{1/3} \ell_s$ as the stadium size already suggests that the minimum size of spatial domains is in general determined by the 11D Planck length $g_s^{1/3} \ell_s$. As a simple but different example, let us consider a measurement of the position of a D-particle inside D4-branes by scattering with an external D-particle. Classically, a bound state of a D-particle with N coincident D4-branes can be described by an instanton solution of the D4-brane gauge field [22]. Probing the D-particle inside these D4-branes is equivalent to probing the localization of the background field A . The localization of A is disturbed by the massless D4-brane open string modes as decay product of pairs of open strings connecting the D-particle probe and D4-branes. We note that D4-brane open string modes are inevitable product if the impact parameter is sufficiently small. The dissipation rate is calculated in [6] for D-particle-D-particle scattering, and in [10] for D-particle-D4-brane scattering. When the impact parameter is of order ΔX , the space-time uncertainty relation shows that the typical energy transferred to the massless open string modes of the D4-brane is of order $\Delta E \sim \Delta T^{-1} \sim \frac{\Delta X}{\ell_s^2}$. If this energy is used for the interaction with the D-particle inside the D4-brane to probe its position, it would contribute to an uncertainty of velocity of the D-particle inside of order $\Delta v = \frac{g_s \ell_s}{v \Delta T}$, which implies uncertainty of the position of order $\frac{g_s \ell_s}{v}$ during the interaction time ΔT . On the other hand, the D-particle inside the D4 brane travels the distance of the order $v \Delta T$. Thus the net uncertainty of the position of the D-particle inside the D4-brane is at least of order

$\Delta Y \sim (\frac{g_s \ell_s}{v} + v \Delta T)/2 \geq \sqrt{\Delta T \ell_s g_s}$ ⁴. Combined with (1), this suggests the validity of a strange relation of the following type, $\Delta X \Delta Y^2 \gtrsim g_s \ell_s^3$. When the D-particle-4-brane impact parameter ΔX is very small, a large lump of massless 4-brane modes is produced and the D-particle inside the 4-branes becomes difficult to locate, $\Delta Y \gtrsim (\frac{g_s \ell_s^3}{\Delta X})^{1/2}$. The 11D Planck scale appeared here again, but the argument is apparently independent of the discussion for the minimum stadium size for the projectile D-particle. We do not know whether this new relation has a universal meaning beyond the situation discussed here. The origin of this relation might be related with the holographic principle [23]. We should note here that, depending on various situations such as the dimensionalities of D-branes and the directions of the spatial distances we are interesting in, the characteristic spatial scale can vary from case to case. This is easily seen from the scaling arguments of the SYM matrix models. We want to emphasize that, in contrast to this, the relation (1) for the transverse distances between D-branes and the time scale is universal since the transverse distances and the time always have opposite scaling behavior. As is seen from the examples discussed above, we expect that most of the space-space relations are derived on the basis of the space-time relation, appropriately combined with the usual quantum mechanical uncertainty principle.

From formal point of view, what is lacking in our arguments is a mathematical foundation for the appearance of the uncertainties. For example, we cannot at present give any precise quantitative definitions for the space and time uncertainties. We feel that there should be a formulation of the theory which surpasses strings and D-branes. In this respect, a very urgent and challenging problem seems to construct a completely covariant formulation of interacting D-particle dynamics as an extension of the SYM models towards its quantum-geometrical reformulation. Covariance would require us to treat even the time variables as noncommutative objects and also to include all brane-antibrane dynamics automatically. It

⁴This argument can be made more accurate by using the distribution $\int p^3 dp$ for a localized wave function, the result will remain the same.

would hopefully lay the mathematical foundation for the space-time uncertainty relation.

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